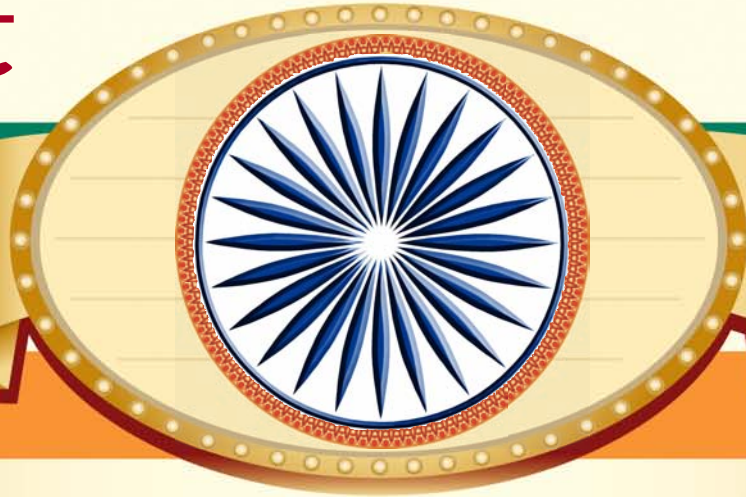


इंटरनेट

मानक



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Whereas the Parliament of India has set out to provide a practical regime of right to information for citizens to secure access to information under the control of public authorities, in order to promote transparency and accountability in the working of every public authority, and whereas the attached publication of the Bureau of Indian Standards is of particular interest to the public, particularly disadvantaged communities and those engaged in the pursuit of education and knowledge, the attached public safety standard is made available to promote the timely dissemination of this information in an accurate manner to the public.

“जानने का अधिकार, जीने का अधिकार”

Mazdoor Kisan Shakti Sangathan

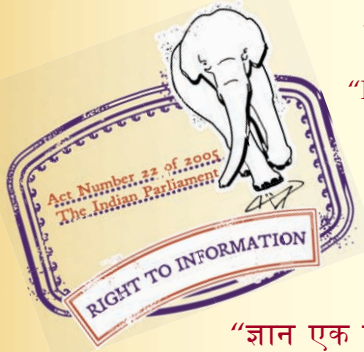
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IS 6200-2 (2004): Statistical Tests of Significance, Part 2: X2- Test [MSD 3: Statistical Methods for Quality and Reliability]



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Bhartrhari—Nitiśatakam

“Knowledge is such a treasure which cannot be stolen”





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भारतीय मानक  
सार्थकता के लिए सांख्यिकीय परीक्षण  
भाग 2  $\chi^2$ -परीक्षण  
( दूसरा पुनरीक्षण )

*Indian Standard*  
STATISTICAL TESTS OF SIGNIFICANCE  
PART 2  $\chi^2$ -TEST  
( *Second Revision* )

ICS 03.120.30

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**BUREAU OF INDIAN STANDARDS**  
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## FOREWORD

This Indian Standard ( Second Revision ) was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

This standard was originally published in 1971 and covered the industrial applications of three main tests of significance, namely,  $t$ -test,  $F$ -test and  $\chi^2$ -test. It was then revised in 1977 into four parts to include tests for normality and also some non-parametric tests, which have wide application in industry.

This second revision of the standard has been undertaken to:

- a) include example for  $2 \times 2$  contingency table,
- b) modify the example for testing goodness of fit, and
- c) incorporate many technical and editorial corrections.

In addition to this the other parts in this series are:

Part 1 Normal,  $t$ -, and  $F$ - tests

Part 3 Tests for normality

Part 4 Non-parametric tests

The composition of the Committee responsible for formulation of this standard is given in Annex B.

*Indian Standard*

## STATISTICAL TESTS OF SIGNIFICANCE

PART 2  $\chi^2$ -TEST*( Second Revision )***1 SCOPE**

This standard lays down application of  $\chi^2$ -test for:

- a) testing of population variance against specified value,
- b) testing for goodness of fit by comparing the observed frequencies with the theoretical or expected frequencies, and
- c) testing for independence in the case of contingency tables.

Each test has been illustrated with the help of examples.

**2 REFERENCES**

The following standards contain provisions, which through reference in this text constitute provision of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

<i>IS No.</i>	<i>Title</i>
6200 ( Part 3 ) : 1984	Statistical tests of significance: Part 3 Tests for normality ( <i>third revision</i> )
7920 ( Part 1 ) : 1994	Statistical vocabulary and symbols Probability and general statistical terms ( <i>second revision</i> )
( Part 2 ) : 1994	Statistical quality control ( <i>second revision</i> )
9300 ( Part 1 ) : 1979	Statistical models for industrial applications : Part 1 Discrete models

**3 TERMINOLOGY**

For the purpose of this standard, the definitions given in IS 7920 ( Part 1 ) and IS 7920 ( Part 2 ) shall apply.

**4 BASIC CONCEPTS**

**4.1** Statistical tests of significance are important tools in decision-making. They are extremely useful in testing the hypothesis of the population variance,

that is, whether a given sample could have arisen from a specified population with the known variability. Thus, if the variability associated with a quality characteristic of product is known beforehand, it is possible to examine whether any observed change in the variability is due to the change in the process parameters or due to chance. In other cases, it may be desirable to test the goodness of fit of a theoretical distribution, say a normal or binomial distribution, to a given set of observations. In yet other cases, it may be necessary to find out whether any meaningful association exists between observations obtained under different classifications. Thus it may be desirable to find out whether the tyre wear of an automobile depends on the location of the tyre or whether the production of non-conforming pieces depends on the shift, machine, process and material.

**4.2 Formulation of Hypotheses**

For taking a decision using statistical tests of significance, the first step is to form the hypotheses, namely, Null Hypothesis (  $H_0$  ) and Alternative Hypothesis (  $H_1$  ).

**4.2.1 Null Hypothesis (  $H_0$  )**

The procedure commonly used is to first set up a null hypothesis regarding equivalence ( no difference ). The question on which the decision is called for, by applying the tests of significance, is translated in terms of null hypothesis in such a way that this null hypothesis would likely to be rejected if there is enough evidence against it as seen from the data in the sample. For example, in the case of new machine, a null hypothesis will be that there is no significant decrease in variation or the data follows the specified distribution.

**4.2.2 Alternative Hypothesis (  $H_1$  )**

Alternative hypothesis is a hypothesis that will be preferred in case the null hypothesis is not true.

**4.3 Level of Significance**

**4.3.1** There are two kinds of errors involved in taking the decision based on the tests of significance, namely:

- a) *Type I error* — Error in deciding that a

significant difference exists when there is no real difference.

- b) *Type II error* — Error in deciding that no difference exists when there is a real difference.

**4.3.2** Type I error and Type II error are also called error of the first kind and error of the second kind respectively. This process of decision making is described in the Table 1.

**Table 1 Process of Decision Making**

	$H_0$ True	$H_1$ True
Reject $H_0$	Type I error	Correct decision
Accept $H_0$	Correct decision	Type II error

**4.3.3** Based on the distribution of test statistics used, it is possible to work out the probability of committing Type I error. The probability of committing Type I error is called level of significance ( $\alpha$ ). The probability of committing Type II error is called level of significance ( $\beta$ ). It is not possible to minimize both these probabilities (risk) at the same time. Hence, assigning to it a chosen level of probability controls one of the risks, usually of the first kind. Generally the value for level of significance is chosen as 0.05 or 0.01, that is, 5 percent or 1 percent. This implies confidence level of 95 percent or 99 percent respectively.

**4.4** The decision-making procedure involves the comparison of the calculated value of the  $\chi^2$  with the tabulated value. The tabulated values of  $\chi^2$  at 5 percent and 1 percent level of significance are given in Annex A. These values will be used in taking the decision. If the calculated value is greater than or equal to the tabulated value of the  $\chi^2$ , then  $H_0$  is rejected, thereby accepting  $H_1$ ; otherwise  $H_0$  is not rejected. For practical purpose,  $H_0$  not rejected is taken as if it is accepted.

**4.5** For each test of significance, certain underlying assumptions are made ( see 5.2, 6.2, 7.2 and 8.2 ). Hence, it is important that these tests are not used indiscriminately. If the assumptions are in doubt, it is advisable to obtain the guidance of a competent statistician to ascertain the feasibility of application of these tests.

**4.6**  $\chi^2$ -tests may either be exact, valid for any sample size, or approximate, valid only for large samples. Exact  $\chi^2$ -tests are discussed in 5 while large sample  $\chi^2$ -tests have been discussed in 6 to 8.

**5 TESTING OF POPULATION VARIANCE AGAINST SPECIFIED VALUE**

**5.1**  $\chi^2$ -test is used for testing whether the variance of a population differs from the specified value. This

test is valid for any sample size, small or large.

**5.2** It is assumed that the observations follow normal distribution and are drawn at random.

**5.3** In this case, null hypothesis is  $H_0 : \sigma^2 = \sigma_0^2$  and alternative hypothesis is either  $H_1 : \sigma^2 > \sigma_0^2$  or  $H_0 : \sigma^2 < \sigma_0^2$ .

Calculate the sample variance [ $s^2 = \Sigma (x - \bar{x})^2 / n - 1$ ] and compute

$$\chi^2 = (n-1)s^2/\sigma_0^2$$

where  $\chi^2$  has  $(n-1)$  degrees of freedom,  $n$  being the size of sample.

Compare the tabulated values of  $\chi^2$  with the calculated value and take the decision as per 4.4.

**5.4 Examples**

**5.4.1** The population variance for strength of yarn was known from the past data to be 0.110. Twenty specimens were tested from a fresh batch of the product and the variance was found to be 0.248. Does this imply a significant increase in variance?

*Solution*

Here  $H_0 : \sigma^2 = 0.110$  and  $H_1 : \sigma^2 > 0.110$

The value of sample variance calculated from the data is  $s^2 = 0.248$

$$\chi^2 = 19 \times 0.248 / 0.110 = 42.8$$

The tabulated value of  $\chi^2$  for 19 degrees of freedom at 1 percent level of significance, from Annex A, is observed to be 36.19 (upper tail). Since the calculated value is greater than this value, null hypothesis ( $H_0$ ) is rejected, implying thereby that the variance of the fresh batch is significantly greater than the known value of 0.110.

**5.4.2** The variance of life of a certain brand of tube-light was estimated as 2 680 hours. This was considered too high, and some corrective action was taken on the process. Subsequently, a fresh sample of 25 tube-lights was tested and the variance was observed to be 1 865. Do these data indicate that the corrective action was effective in reducing the variance?

*Solution*

Here  $H_0 : \sigma^2 = 2 680$  against  $H_1 : \sigma^2 < 2 680$

The value of sample variance calculated from the data is  $s^2 = 1 865$

$$\chi^2 = 24 \times 1 865 / 2 680 = 16.701, \text{ degrees of freedom (df) } = 24.$$

The tabulated value of  $\chi^2$  for 24 degree of freedom at

5 percent level of significance, that is,  $\chi^2_{0.95; 24}$  from Annex A is 13.85 ( lower tail ). Since the calculated value is larger than this tabulated value,  $H_0$  is rejected at 5 percent level of significance, implying that, the corrective action has been effective in reducing the variance significantly.

## 6 TESTING GOODNESS OF FIT

**6.1**  $\chi^2$ -test is also used for testing for goodness of fit of some theoretical model, that is, whether the frequencies observed for certain categories differ significantly from expected or theoretical frequencies under the model.

**6.2** In this case number of observations should be large ( more than 30 ). It is also important that the expected frequency in any class should not be less than 5. If the expected frequency in any class is less than 5, then it may be pooled with adjacent class so that the expected frequency becomes 5 or more and then the  $\chi^2$ -tests can be applied with little effect on the significance level.

**6.3** If  $o_1, o_2, \dots, o_k$  represent the observed frequencies and  $e_1, e_2, \dots, e_k$  represent the expected frequencies, the test-statistic to be used for the purpose is:

$$\chi^2 = \sum (o_i - e_i)^2 / e_i = \sum o_i^2 / e_i - n$$

where  $n$  = total frequency and  $\chi^2$  has  $(k - 1)$  degrees of freedom or less depending on how the expected frequencies are computed from the data. The degrees of freedom would be further reduced by 1 for each parameter estimated from sample. Thus if  $p$  parameters are estimated, the degrees of freedom would be  $(k - p - 1)$ .

The  $\chi^2$ -test is also useful in finding out whether a theoretical distribution like Binomial, Poisson, Normal or any other distribution, fits the given observations satisfactorily or not. The use of this test for testing if the data follows normal distribution has been explained in IS 6200 ( Part 3 ).

### 6.4 Example

The distribution of 50 samples of 100 items each according to number of non-conforming items observed is given in Table 2. Test if the data follow Poisson distribution.

#### Solution

Here Null hypothesis is,  $H_0$ : The data follow Poisson distribution against the alternative hypothesis,  $H_1$ : The data do not follow Poisson distribution.

From the data, average number of non-conformities per sample  $(\sum f_i x_i / \sum f_i)$  is 1.98 and therefore average proportion of non-conformities per item =  $1.98/100 = 0.0198 \approx 0.02$  as the sample size is 100.

**Table 2 Distribution**  
( Clause 6.4 )

SI No.	No. of Non-conforming Items	No. of Samples
	(x)	(f)
(1)	(2)	(3)
i)	0	8
ii)	1	12
iii)	2	13
iv)	3	9
v)	4	7
vi)	5	0
vii)	6	1
viii)	7	0
	Total	50

The expected frequencies under the assumption that the data follow Poisson distribution are calculated in Table 3.

As the expected frequencies in the last 4 classes are less than 5 each, they are pooled together.  $\chi^2$  is then calculated as:

$$\chi^2 = \sum_{i=1}^5 o_i^2 / e_i^2 - n = 50.55 - 50 = 0.55.$$

This value of  $\chi^2$  is based on 5 cells, but since one parameter, namely, mean =  $\lambda$  has been estimated from the observed data,  $\chi^2$  has  $5 - 1 - 1 = 3$  degree of freedom ( see 6.3 ). From Annex A, we see that the tabulated value of  $\chi^2$  with 3 degree of freedom is 7.82 ( upper tail ), the observed value of  $\chi^2$  being less than the tabulated value, the fit is considered to be good, and the observed data can be taken to follow Poisson distribution.

## 7 TESTING FOR INDEPENDENCE IN CONTINGENCY TABLES

**7.1** The  $\chi^2$ -test is used for testing whether a set of observations classified according to two attributes are associated or not, that is, testing for independence in the case of contingency tables.

**7.2** In this case number of observations should be large ( more than 30 ). It is also important that the expected frequency in any cell should not be less than 5.

**7.3** When a set of observations is tabulated according to two factors in  $r$  rows and  $c$  columns, a two-way table is obtained with  $r \times c$  cells. Such two-way tables are also called contingency tables.

**7.3.1** The null hypothesis is that the two classifications are independent, that is, the probability that an observation falls in a particular row ( column ) is not



**Table 3 Computation for Testing Goodness of Fit**

(Clause 6.4)

No. of Non-conforming Items <i>x</i>	Observed Frequencies <i>o<sub>i</sub></i>	$e^{-\lambda}\lambda^x/x!$	Expected Frequency $e_i = (3) \times 50$	$o_i^2/e_i$
(1)	(2)	(3)	(4)	(5)
0	8	.138 1	6.9	9.28
1	12	.273 4	13.7	10.51
2	13	.270 6	13.5	12.52
3	9	.178 6	8.9	9.10
4	7	.088 4	4.4	7.0 9.14
5	0	.035 0	1.8	
6	1	.011 6	0.6	
7	0	.004 3	0.2	
Total	50	1.000 0		50.55

affected by the particular column ( row ) to which it belongs. If the null hypothesis is rejected, the two attributes of classification are said to be dependent or correlated.

Here null hypothesis is  $H_0: p_{ij} = p_i \times p_j$  and alternative hypothesis is  $H_1: p_{ij} \neq p_i \times p_j$

where

$p_{ij}$  = probability that an item will fall in ( *i, j* )th cell

$p_i$  = probability that an item will fall in *i*th row

$p_j$  = probability that an item will fall in *j*th column

**7.3.2** If  $o_{11}, o_{12} \dots o_{rc}$  represent the observed frequencies in the *rc* cells of the contingency table, then the expected frequencies  $e_{11}, e_{12} \dots e_{rc}$  corresponding to the *rc* cells are obtained with the help of the marginal totals and the overall total. Thus the expected frequency,  $e_{ij}$  is calculated as under:

$$e_{ij} = R_i \times C_j / n$$

where

$R_i$  = *i*th row total, and

$C_j$  = *j*th column total, and

$n$  = overall or grand total.

The test statistic is then calculated as to make the procedure more user-friendly, the simplified form of  $\chi^2$ , namely.

$$\chi^2 = n \left\{ \sum_{i=1}^r \sum_{j=1}^c o_{ij}^2 / (R_i \times C_j) - 1 \right\} = \sum_{i=1}^r \sum_{j=1}^c o_{ij}^2 / e_{ij} - n$$

where

$$R_i = \sum_{j=1}^c o_{ij} \quad C_j = \sum_{i=1}^r o_{ij} \quad n = \sum_{i=1}^r \sum_{j=1}^c o_{ij}$$

**7.4 Example**

During a certain period, the number of breakdowns of 4 machines occurring in each of the 3 shifts was recorded which is summarized in the Table 4.

It is intended to examine whether the same percentage

**Table 4 Number of Breakdowns Shiftwise for Four Machines**

( Clause 7.4 )

Shift	Number of Breakdowns Machine				Total
	A	B	C	D	
(1)	(2)	(3)	(4)	(5)	(6)
1	10 ( 8.6 )	6 ( 7.3 )	12 ( 11.5 )	13 ( 13.6 )	41
2	10 ( 13.0 )	12 ( 11.1 )	19 ( 17.4 )	21 ( 20.5 )	62
3	13 ( 11.4 )	10 ( 9.6 )	13 ( 15.1 )	18 ( 17.9 )	54
Total	33	28	44	52	157

of breakdown occurs on each machine during each shift.

Here  $H_0$  : Breakdowns in machines are independent of shifts.

$H_1$  : Breakdowns in machines are dependent on shifts.

If breakdowns in machines are independent of shifts then the expected number of breakdowns for machine  $A$  in the first shift is calculated as  $(41 \times 33)/157 = 8.6$ . The expected values of breakdown for various combinations of machines and shifts are given in brackets in the above table. The  $\chi^2$  statistic is then calculated as:

$$\chi^2 = 10^2/8.6 + \dots + 18^2/17.9 - 157 = 1.96$$

Since this calculated value is less than the tabulated value of 12.59 [upper tail 5 percent value of  $\chi^2$  distribution given in Annex A corresponding to  $(3-1)(4-1) = 6$  degrees of freedom], the null hypothesis is not rejected, implying breakdowns in machines are independent of shifts.

7.5 If each of the two attributes under consideration is classified into two categories then a  $2 \times 2$  contingency table is obtained. The  $\chi^2$  statistic with one degree of freedom is calculated for the  $2 \times 2$  contingency table as explained in 7.3. The expression so obtained when simplified will be reduced to:

$$\chi^2 = \frac{n(o_{11} o_{22} - o_{12} o_{21})^2}{(o_{11} + o_{12})(o_{21} + o_{22})(o_{11} + o_{21})(o_{12} + o_{22})}$$

where

$$n = o_{11} + o_{12} + o_{21} + o_{22}$$

7.5.1 If some expected value for any of the cell is small,  $\chi^2$  approximation could be considerably improved by applying Yates' correction. This is done by adding  $\frac{1}{2}$  to the smallest observed frequency and keeping the marginal totals the same. Thus the corrected statistic  $\chi^2$  is obtained as:

$$\chi^2 = \frac{n[|o_{11} o_{22} - o_{12} o_{21}| - n/2]^2}{(o_{11} + o_{12})(o_{21} + o_{22})(o_{11} + o_{21})(o_{12} + o_{22})}$$

### 7.5.2 Example

Items produced in a factory were inspected for 2 types of non-conformities  $A$  and  $B$ . Inspection results are given in Table 5.

Examine if the two types of non-conformities develop independently of each other in the product

$$p_{11} = 0.0765$$

$$p_{10} \times p_{01} = 0.2176 \times 0.1397 = 0.0304$$

Solution

Here  $H_0$  : Non-conformity  $A$  develops independently of defect  $B$

$H_1$  : Non-conformities  $A$  and  $B$  do not develop independently of each other.

The observed value of  $\chi^2$  is

$$\chi^2 = \frac{680(52 \times 489 - 96 \times 43)^2}{(95 \times 585 \times 148 \times 532)} = 70.504$$

Tabulated value of  $\chi^2$  for 1 degree of freedom at 5 percent level of significance as given in Annex A is 6.64 (upper tail). Since the calculated value is more than the tabulated value,  $H_0$  is rejected and it is concluded that the two types of non-conformities do not develop independently of each other.

## 8 TESTING FOR HOMOGENEITY OF SEVERAL POPULATIONS

8.1 The  $\chi^2$ -test can be used to test, if several populations are homogeneous in respect of an attribute having several categories.

8.2 This, too, is a large sample test, and will be valid only if total number of observations is large, say more than 30, and expected frequency in each cell should be at least 5.

8.3 If there are  $r$  populations each classified according to  $c$  categories of an attribute, and  $p_{ij}$  = probability of an observation from the  $i$ th population to belong to  $j$ th category, we have to test the hypothesis

$H_0$  :  $p_{1j} = p_{2j} = \dots = p_{kj}$  for all  $j = 1, 2, \dots, c$ . against  
 $H_1$  : Not all these probabilities are equal.

**Table 5 Distribution of Manufactured Items According to Non-conformities  $A$  and  $B$**

( Clause 7.5.2 )

Non-conformity B Present	Non-conformity A Present		Total
	Yes	No	
(1)	(2)	(3)	(4)
Yes	52	96	148
No	43	489	432
Total	95	585	680

If each of  $r$  samples of sizes  $n_1, n_2, \dots, n_r$  taken from  $r$  populations is classified according to  $c$  categories with  $O_{ij}$  as the frequency of the  $(i, j)$ th cell and  $f_j$  as the total frequency for the  $j$ th category, then the expected frequencies  $e_{ij}$ 's are calculated as  $e_{ij} = n_i \times f_j / n$ , where  $n = n_1 + n_2 + \dots + n_r$ . The expression for  $\chi^2$  is as in 7.3.2.

8.4 For both 7 and 8, if we have a  $2 \times k$  table as:

Row	Column		Total
	1	2	
1	$a_1$	$b_1$	$T_1$
2	$a_2$	$b_2$	$T_2$
:	:	:	
$k$	$a_k$	$b_k$	$T_k$
Total	$T_a$	$T_b$	$n$

the simplified form of  $\chi^2$  is:

$$\chi^2 = n^2 / (T_a \times T_b) [ \sum_{i=1}^k (a_i^2 / T_i) - T_a^2 / n ]$$

with  $(k - 1) \times (2 - 1) = k - 1$  degree of freedom.

8.5 Examples

8.5.1 Two sets of data on non-conformities of a

process were collected — one for identifying the major causes through a Pareto analysis, and the other after some corrective actions for improvement were initiated. The data are given in Table 6. Examine if the corrective actions were effective in reducing the major causes of non-conformities .

Solution

Here  $H_0$ : Corrective actions for improvement have not been effective, that is, the two sets of data come from identical distributions over non-conformities.

Against  $H_1$ : These actions have been effective.

Using the formula for  $\chi^2$  given in 8.4, we get  $\chi^2 = 289^2 / (181 \times 108) [ 116.79 - 181^2 / 289 ] = 14.66$  with degree of freedom =  $(6 - 1) \times (2 - 1) = 5$ . The tabulated value of  $\chi^2$  with 5 degree of freedom at 5 percent level of significance is obtained as 11.07 ( upper tail ) from Annex A. As the observed value is higher than this tabulated value we conclude that  $H_0$  is rejected, that is, the corrective actions for improvement have been effective to some extent.

8.5.2 In a mass production process, a sample of 1 000 items was taken from each day's production and the number of non-conforming items was obtained as given in Table 7.

Table 6 Data on Non-conformities of a Process Before and After Improvement

( Clause 8.5.1 )

Causes of Non-conformities	Number of Non-conforming Cases		Total $T_i$	$a_i^2 / T_i$
	Before	After		
	Improvement $a_i$	Improvement $b_i$		
(1)	(2)	(3)	(4)	(5)
Improper rotation	68	24	92	50.26
Noise	39	18	57	26.68
Wobble	32	20	52	19.69
Pressure	16	17	33	7.76
Left over	14	13	27	7.26
Others	12	16	28	5.14
Total	181 ( $T_a$ )	108 ( $T_b$ )	289 ( $n$ )	116.79

Table 7 Data from a Mass Production Process

( Clause 8.5.2 )

Day	No. of Non-conformities	No. of Conformities
(1)	(2)	(3)
First	12	988
Second	16	984
Third	8	992
Fourth	14	986
Fifth	10	990
Total	60	4 940

It is required to test the hypothesis that the proportion of non-conforming is constant from day to day.

*Solution*

Using the formula for  $\chi^2$  given in 8.4 we get:

$$\chi^2 = [5000^2 / (60 \times 4940)] [(12^2 + 16^2 + 8^2 + 14^2 + 10^2) / 1000 - 60^2 / 5000] = 3.374 \text{ with } (5 - 1) \times$$

$$(2 - 1) = 4 \text{ degrees of freedom}$$

This value is less than the tabulated value of 9.49 (upper tail), which is the 5 percent value of the  $\chi^2$  distribution with 4 degree of freedom (see Annex A). Since  $\chi^2$  is not significant the data do not indicate that the proportion of non-conforming varies from day to day.

## ANNEX A

( Clauses 4.4, 5.4.1, 5.4.2 , 6.4, 7.4, 7.5.2, 8.5.1 and 8.5.2 )

CRITICAL VALUES OF  $\chi^2$ -DISTRIBUTION

Degree(s) of Freedom	Significance Level ( Upper Tail )		Significance Level ( Lower Tail )	
	0.05	0.01	0.05	0.01
(1)	(2)	(3)	(4)	(5)
1.	3.84	6.64	0.000 16	0.003 9
2.	5.99	9.21	0.02	0.10
3.	7.82	11.34	0.11	0.35
4.	9.49	13.28	0.3	0.71
5.	11.07	15.09	0.55	1.15
6.	12.59	16.81	0.87	1.64
7.	14.07	18.48	1.24	2.17
8.	15.51	20.09	1.65	2.73
9.	16.92	21.67	2.09	3.33
10.	18.31	23.21	2.56	3.94
11.	19.68	24.73	3.05	4.50
12.	21.03	26.22	3.57	5.23
13.	22.36	27.69	4.11	5.89
14.	23.69	29.14	4.66	6.57
15.	25.00	30.58	5.23	7.26
16.	26.30	32.00	5.81	7.96
17.	27.59	33.41	6.41	8.67
18.	28.87	34.81	7.01	9.39
19.	30.14	36.19	7.63	10.12
20.	31.41	37.56	8.26	10.85
21.	32.67	38.93	8.90	11.59
22.	33.92	40.29	9.54	12.34
23.	35.17	41.64	10.20	13.09
24.	36.42	42.98	10.86	13.85
25.	37.65	44.31	11.52	14.64
26.	68.89	45.64	12.20	15.38
27.	40.11	46.96	12.88	16.15
28.	41.34	48.28	13.56	16.93
29.	42.56	49.59	14.26	17.71
30.	43.77	50.89	14.95	18.49
40.	55.75	63.69	22.16	26.51
50.	67.50	76.15	29.71	34.77
60.	79.18	88.38	37.8	43.19
70.	90.53	100.42	45.44	51.74
80.	101.88	112.33	53.54	60.39
90.	113.14	124.12	61.75	69.13
100.	124.34	135.81	70.06	77.93

## ANNEX B

## ( Foreword )

## COMMITTEE COMPOSITION

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*Member-Secretary*

SHRI LALIT KUMAR MEHTA  
Joint Director ( MSD ), BIS

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